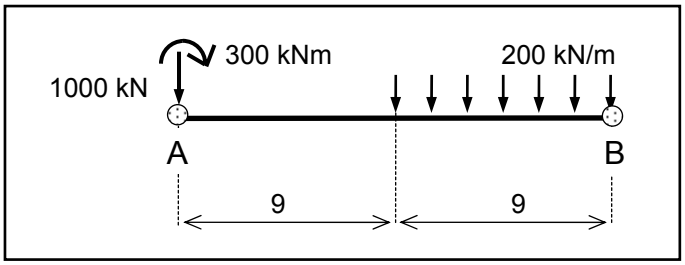
PRZYKŁAD LICZBOWY

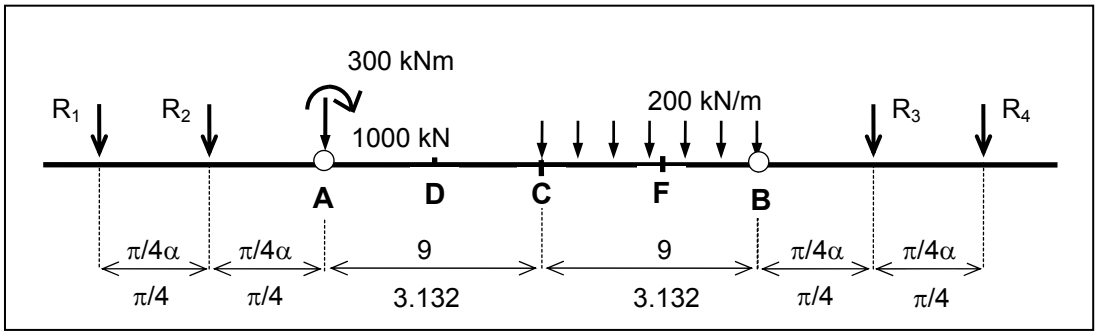


b = 1.6 m

h = 0.8 m

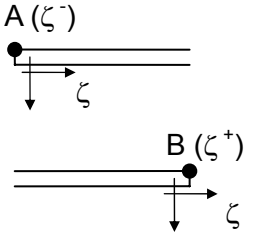
c = 60 MPa/m

$E = 2.4 \times 10^4$ MPa



$$\alpha = \sqrt[4]{\frac{bc}{4EJ}} = \sqrt[4]{\frac{1.6 \times 60 \times 10^6 \times 12}{4 \times 2.4 \times 10^{10} \times 1.6 \times 0.8^3}} = 0.348 \quad [1/m]$$

* warunki zgodności, obliczenie reakcji



1) $M_A^{pr} = 300\ \text{kNm} = M$

2) $Q_A^{pr} = -1000\ \text{kN} = -P$

3) $M_B^{lew} = 0$

4) $Q_B^{lew} = 0$

1) $$\frac{1}{4\alpha}\left[R_1\,\eta_1\left(-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-\frac{\pi}{4}\right) + P\,\eta_1(0) + R_3\,\eta_1\left(6.264+\frac{\pi}{4}\right) + R_4\,\eta_1\left(6.264+\frac{\pi}{2}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{3.132}^{6.264} + \frac{1}{2}(-1)M\,\eta_2(0) = M$$

2) $$-\frac{1}{2}\left[R_1(-1)\,\eta_2\left(-\frac{\pi}{2}\right) + R_2(-1)\,\eta_2\left(-\frac{\pi}{4}\right) + P(-1)\,\eta_2(0) + R_3\,\eta_2\left(6.264+\frac{\pi}{4}\right) +\right.$$
$$\left. + R_4\,\eta_2\left(6.264+\frac{\pi}{2}\right)\right] + \left(-\frac{q}{4\alpha}\left[\eta_1\right]_{3.132}^{6.264}\right) - \frac{\alpha}{2}M\,\eta(0) = -P$$

3) $$\frac{1}{4\alpha}\left[R_1\,\eta_1\left(-6.264-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-6.264-\frac{\pi}{4}\right) + P\,\eta_1(-6.264) + R_3\,\eta_1\left(\frac{\pi}{4}\right) + R_4\,\eta_1\left(\frac{\pi}{2}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_0^{|-3.132|} + \frac{1}{2}(-1)M\,\eta_2(-6.264) = 0$$

4) $$-\frac{1}{2}\left[R_1(-1)\,\eta_2\left(-6.264-\frac{\pi}{2}\right) + R_2(-1)\,\eta_2\left(-6.264-\frac{\pi}{4}\right) + P(-1)\,\eta_2(-6.264) +\right.$$
$$\left. + R_3\,\eta_2\left(\frac{\pi}{4}\right) + R_4\,\eta_2\left(\frac{\pi}{2}\right)\right] + \left(-(-1)\frac{q}{4\alpha}\left[\eta_1\right]_0^{|-3.132|}\right) - \frac{\alpha}{2}M\,\eta(-6.264) = 0$$

1) $\frac{1}{4\alpha}\left[-0.2079R_1+0+1000+0+0\right]+\frac{200}{4\times 0.348^2}\left[0-0\right]+\frac{300}{2}-300=0$

2) $-\frac{1}{2}\left[0+0.3224R_2+1000+0+0\right]-\frac{200}{4\times 0.348}\left[0.00181+0.04321\right]-\frac{300\times 0.348}{2}+1000=0$

3) $\frac{1}{4\alpha}\left[0+0+0+0-0.2079R_4\right]+\frac{200}{4\times 0.348^2}\left[0-0\right]+150\times 0.00184=0$

4) $-\frac{1}{2}\left[0+0+0-0.3224R_3+0\right]+\frac{200}{4\times 0.348}\left[-0.04321-1\right]-\frac{300\times 0.348}{2}0.00187=0$

$$-0.1493R_1=-568.4 \quad\Rightarrow\quad R_1=3807\ \text{kN}$$
$$-0.1612R_2=-441.3 \quad\Rightarrow\quad R_2=2738\ \text{kN}$$
$$-0.1494R_4=-0.276 \quad\Rightarrow\quad R_4\cong 0\ \text{kN}$$
$$0.1612R_3=150 \quad\Rightarrow\quad R_3=930\ \text{kN}$$

* **ugięcia**

$\frac{\alpha}{2k}=1.8125\times 10^{-6}$ [m / kN] $\qquad \frac{q}{2k}=1.0417\times 10^{-3}$ [m] $\qquad \frac{M\alpha^2}{k}=3.7845\times 10^{-4}$ [m]

$$w_A^{pr}=\frac{\alpha}{2k}\left[R_1\eta\left(-\frac{\pi}{2}\right)+R_2\eta\left(-\frac{\pi}{4}\right)+P\eta(0)+R_3\eta\left(6.264+\frac{\pi}{4}\right)\right]+$$
$$+\frac{q}{2k}\left[\eta_2\right]_{3.132}^{6.264}-\frac{\alpha^2}{k}(-1)M\eta_3(0)=6.402\ \text{[mm]}$$

$$w_D=\frac{\alpha}{2k}\left[R_1\eta\left(-1.566-\frac{\pi}{2}\right)+R_2\eta\left(-1.566-\frac{\pi}{4}\right)+P\eta(-1.566)+R_3\eta\left(4.698+\frac{\pi}{4}\right)\right]+$$
$$+\frac{q}{2k}\left[\eta_2\right]_{1.566}^{4.698}-(-1)\frac{M\alpha^2}{k}\eta_3(-1.566)=0.158\ \text{[mm]}$$

$$w_C=\frac{\alpha}{2k}\left[R_1\eta\left(-3.132-\frac{\pi}{2}\right)+R_2\eta\left(-3.132-\frac{\pi}{4}\right)+P\eta(-3.132)+R_3\eta\left(3.132+\frac{\pi}{4}\right)\right]+$$
$$+\frac{q}{2k}\left[\eta_2\right]_{0}^{3.132}-(-1)\frac{M\alpha^2}{k}\eta_3(-3.132)=0.755\ \text{[mm]}$$

$$w_F=\frac{\alpha}{2k}\left[R_1\eta\left(-4.698-\frac{\pi}{2}\right)+R_2\eta\left(-4.698-\frac{\pi}{4}\right)+P\eta(-4.698)+R_3\eta\left(1.566+\frac{\pi}{4}\right)\right]+$$
$$+\frac{q}{2k}\left[\eta_2\right]_{0}^{1.566}\times 2-(-1)\frac{M\alpha^2}{k}\eta_3(-4.698)=2.08\ \text{[mm]}$$

$$w_B=\frac{\alpha}{2k}\left[R_1\eta\left(-6.264-\frac{\pi}{2}\right)+R_2\eta\left(-6.264-\frac{\pi}{4}\right)+P\eta(-6.264)+R_3\eta\left(\frac{\pi}{4}\right)\right]+$$
$$+\frac{q}{2k}\left[\eta_2\right]_{0}^{|-3.132|}+\frac{M\alpha^2}{k}\eta_3(-6.264)=2.183\ \text{[mm]}$$

* **momenty zginające**

$\frac{1}{4\alpha}=0.7184$ [m] $\qquad \frac{q}{4\alpha^2}=412.868$ [kNm]

$$M_A^{pr} = \frac{1}{4\alpha}\left[R_1\,\eta_1\left(-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-\frac{\pi}{4}\right) + P\,\eta_1(0) + R_3\,\eta_1\left(6.264+\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{3.132}^{6.264} - \frac{M}{2}\eta_2(0) = 299.8\ [\text{kNm}]$$

$$M_D = \frac{1}{4\alpha}\left[R_1\,\eta_1\left(-1.566-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-1.566-\frac{\pi}{4}\right) + P\,\eta_1(-1.566) + R_3\,\eta_1\left(4.698+\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{1.566}^{4.698} - \frac{M}{2}\eta_2(-1.566) = 616.8\ [\text{kNm}]$$

$$M_C = \frac{1}{4\alpha}\left[R_1\,\eta_1\left(-3.132-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-3.132-\frac{\pi}{4}\right) + P\,\eta_1(-3.132) + R_3\,\eta_1\left(3.132+\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{0}^{3.132} - \frac{M}{2}\eta_2(-3.132) = -14.12\ [\text{kNm}]$$

$$M_F = \frac{1}{4\alpha}\left[R_1\,\eta_1\left(-4.698-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-4.698-\frac{\pi}{4}\right) + P\,\eta_1(-4.698) + R_3\,\eta_1\left(1.566+\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{0}^{1.566} \times 2 - \frac{M}{2}\eta_2(-4.698) = 105.79\ [\text{kNm}]$$

$$M_B = \frac{1}{4\alpha}\left[R_1\,\eta_1\left(-6.264-\frac{\pi}{2}\right) + R_2\,\eta_1\left(-6.264-\frac{\pi}{4}\right) + P\,\eta_1(-6.264) + R_3\,\eta_1\left(\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha^2}\left[\eta_3\right]_{0}^{|-3.132|} - \frac{M}{2}\eta_2(-6.264) = 0\ [\text{kNm}]$$

* **siły poprzeczne**

$$\frac{q}{4\alpha} = 143.678\ [\text{kN}]$$

$$Q_A^{pr} = -\frac{1}{2}\left[-R_1\,\eta_2\left(-\frac{\pi}{2}\right) - R_2\,\eta_2\left(-\frac{\pi}{4}\right) - P\,\eta_2(0) + R_3\,\eta_2\left(6.264+\frac{\pi}{4}\right)\right] +$$
$$- \frac{q}{4\alpha}\left[\eta_1\right]_{3.132}^{6.264} - \frac{M\alpha}{2}\eta(0) = -1000\ [\text{kN}]$$

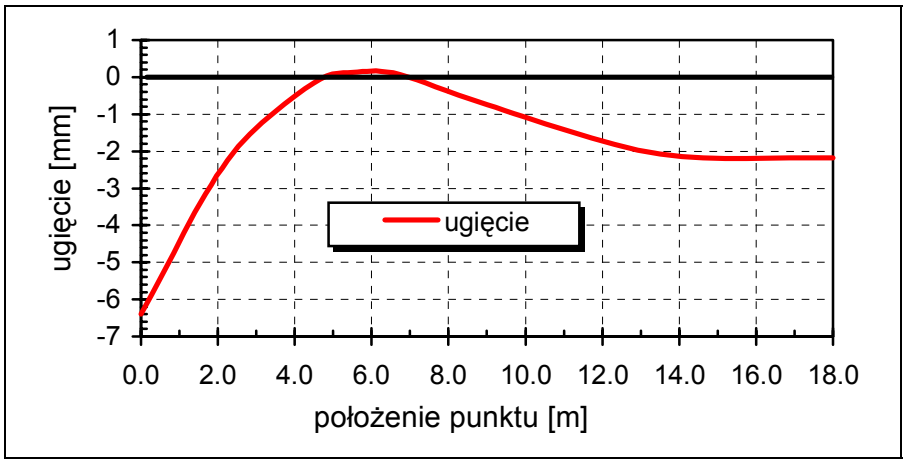
$$Q_D = -\frac{1}{2}\left[-R_1\,\eta_2\left(-1.566-\frac{\pi}{2}\right) - R_2\,\eta_2\left(-1.566-\frac{\pi}{4}\right) - P\,\eta_2(-1.566) + R_3\,\eta_2\left(4.698+\frac{\pi}{4}\right)\right] +$$
$$- \frac{q}{4\alpha}\left[\eta_1\right]_{1.566}^{4.698} - \frac{M\alpha}{2}\eta(-1.566) = 133.3\ [\text{kN}]$$

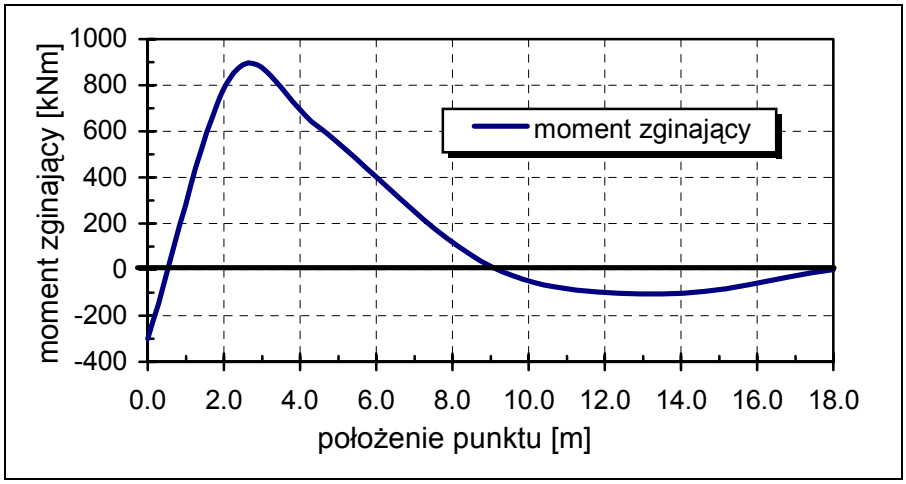
$$Q_C = -\frac{1}{2}\left[-R_1\,\eta_2\left(-3.132-\frac{\pi}{2}\right) - R_2\,\eta_2\left(-3.132-\frac{\pi}{4}\right) - P\,\eta_2(-3.132) + R_3\,\eta_2\left(3.132+\frac{\pi}{4}\right)\right] +$$
$$- \frac{q}{4\alpha}\left[\eta_1\right]_{0}^{3.132} - \frac{M\alpha}{2}\eta(-3.132) = 187.24\ [\text{kN}]$$

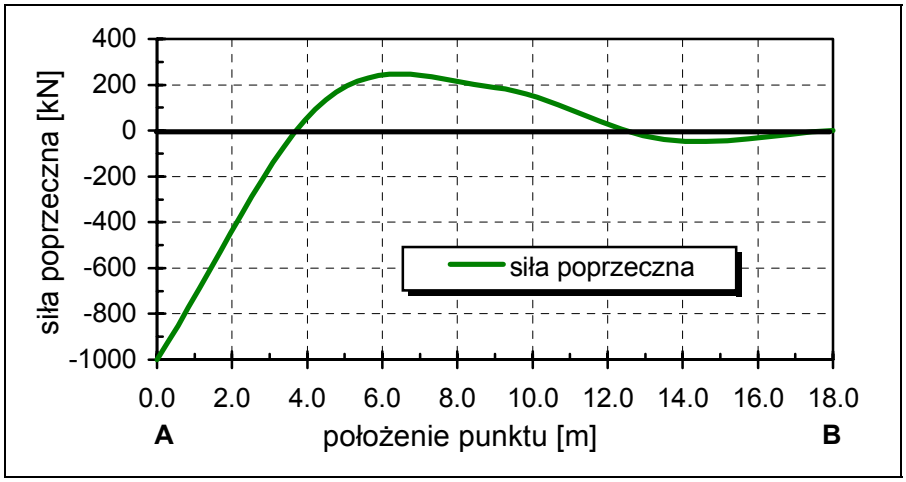
$$Q_F = -\frac{1}{2}\left[-R_1\,\eta_2\left(-4.698-\frac{\pi}{2}\right) - R_2\,\eta_2\left(-4.698-\frac{\pi}{4}\right) - P\,\eta_2(-4.698) + R_3\,\eta_2\left(1.566+\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha}\left[\eta_1\right]_{0}^{|-1.566|} - \frac{q}{4\alpha}\left[\eta_1\right]_{0}^{1.566} - \frac{M\alpha}{2}\eta(-4.698) = -38.09\ [\text{kN}]$$

$$Q_B = -\frac{1}{2}\left[-R_1\,\eta_2\left(-6.264-\frac{\pi}{2}\right) - R_2\,\eta_2\left(-6.264-\frac{\pi}{4}\right) - P\,\eta_2(-6.264) + R_3\,\eta_2\left(\frac{\pi}{4}\right)\right] +$$
$$+ \frac{q}{4\alpha}\left[\eta_1\right]_{0}^{|-3.132|} - \frac{M\alpha}{2}\eta(-6.264) = 0\ [\text{kNm}]$$

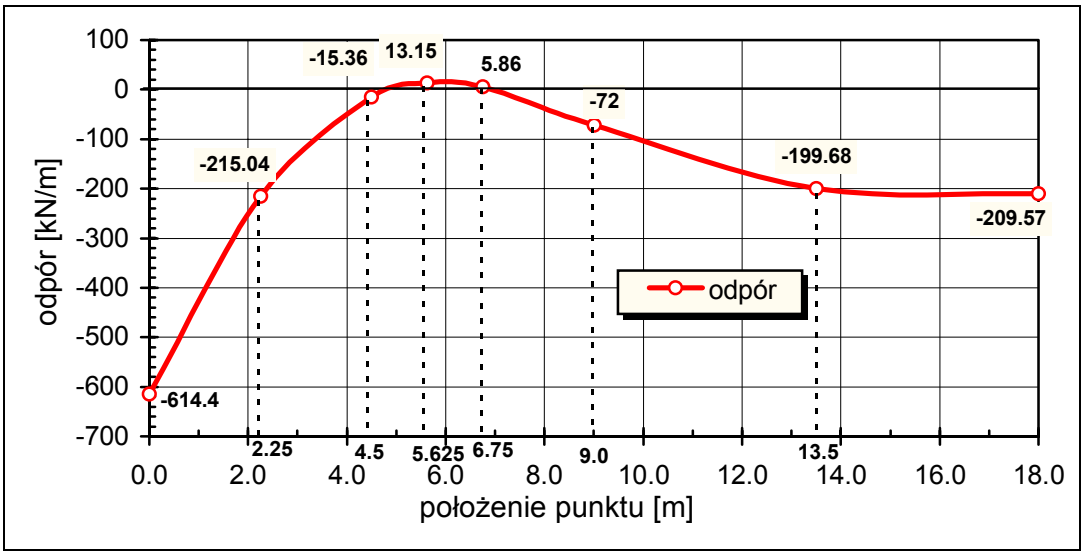
* **wykresy linii ugięcia, momentów zginających i sił poprzecznych**







* **wykres odporu, sprawdzenie równowagi**



$$r = b \times c \times w = 1.6\ [m] \times 60\ [MPa / m] \times w\ [mm] = 96 \times w\ [kN / m]$$

* **suma sił** (aproksymacja pola trapezami)

$$\sum r = 0.5 \times (614.4 + 215.04) \times 2.25 + 0.5 \times (215.04 + 15.36) \times 2.25 + 0.5 \times 15.36 \times 1.125 -$$
$$- 0.5 \times 13.15 \times 1.125 + 0.5 \times (13.15 + 5.86) \times 1.125 + 0.5 \times 72.0 \times 2.25 - 0.5 \times 5.86 \times 2.25 +$$
$$+ 0.5 \times (199.68 + 72.0) \times 4.5 + 0.5 \times (199.68 + 209.57) \times 4.5 = 2810.8\ [kN]$$

$$\sum P = 200 \times 9 + 1000 = 2800\ [kN]$$

$$\text{błąd } \Delta = \frac{\left(\sum P - \sum r\right)}{\sum P} \times 100\% \approx 0.4\%$$

* **suma momentów wzg. B** (aproksymacja pola trójkątami i prostokątami)

$$\sum M_r = 0.5 \times 399 \times 2.25 \times (2 / 3 \times 2.25 + 15.75) + 215.04 \times 2.25 \times 16.875 + 15.36 \times 2.25 \times 14.625 +$$
$$+ 0.5 \times 199.68 \times 2.25 \times (2 / 3 \times 2.25 + 13.5) + 0.5 \times 15.36 \times 1.125 \times (12.375 + 2 / 3 \times 1.125) -$$
$$- 0.5 \times 13.15 \times 1.125 \times (1 / 3 \times 1.125 + 12.375) - 5.86 \times 1.125 \times (11.25 + 1.125 / 2) -$$
$$- 0.5 \times 7.29 \times (11.25 + 2 / 3 \times 1.125) + 0.5 \times 72 \times 2.25 \times (9 + 1 / 3 \times 2.25) -$$
$$- 0.5 \times 5.86 \times 2.25 \times (9 + 2 / 3 \times 2.25) + 199.68 \times 4.5 \times 2.25 + 0.5 \times 9.89 \times 4.5 \times 4.5 / 3 +$$
$$+ 72 \times 4.5 \times 6.75 + 0.5 \times 127.68 \times 4.5 \times (4.5 + 1 / 3 \times 4.5) = 26367\ [kNm]$$

$$\sum M_P = -1000 \times 18 + 300 - 200 \times 9 \times 4.5 = -25800\ [kNm]$$

$$\text{błąd } \Delta = \frac{\left|\sum M_P\right| - \sum M_r}{\left|\sum M_P\right|} \times 100\% \approx -2.2\%$$