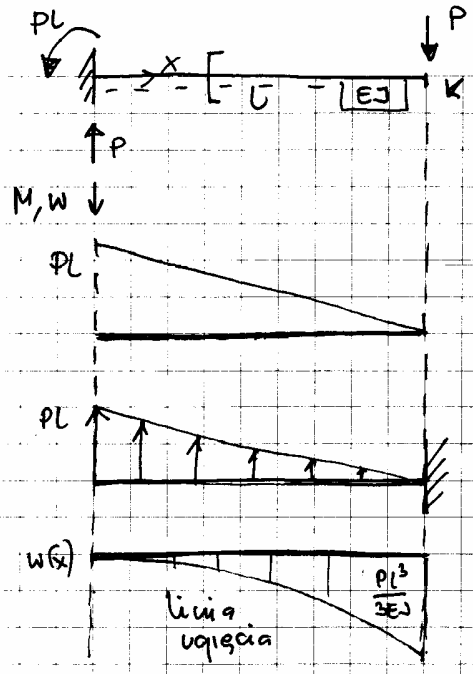
① Narysować ugiętą oś belki. Obliczyć ugięcie końca wspornika



$M(x) = Px - Pl$

$EJw'' = -M(x) = Pl - Px$

$EJw' = A + Plx - Px^2/2$

$EJw = B + Ax + Plx^2/2 - Px^3/6$

$w'(0) = 0 \Rightarrow A = 0$

$w(0) = 0 \Rightarrow B = 0$

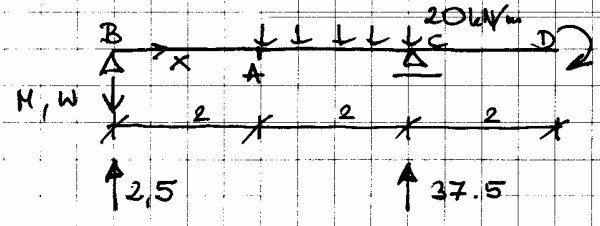
$$w(x) = \frac{P}{2EJ}\left(x^2 l - x^3/3\right)$$

$$w_K = w(l) = \frac{Pl^3}{3EJ}$$

met. Mohra,

$$w_K = \frac{M^P_K}{EJ} = \frac{1}{EJ}\left[\frac{1}{2} Pl \cdot l \cdot \frac{2}{3} l\right] = \frac{Pl^3}{3EJ}$$

② Met. analityczna, obliczyć ugięcie w pkt. A i D osi belki



$$M(x) = 2{,}5x\Big|_{BA} - \frac{20(x-2)^2}{2}\Big|_{AC} + \frac{20(x-4)^2}{2} + 37.5(x-4)\Big|_{CD}$$

$$EJw'' = -2.5x\Big|_{BA} + 10(x-2)^2\Big|_{AC} - 10(x-4)^2 - 37.5(x-4)\Big|_{CD}$$

~~$EJw' = -\frac{2.5x^2}{2}$~~

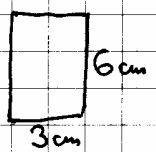
$$EJw' = C_1 - \frac{2.5x^2}{2}\Big|_{BA} + \frac{10(x-2)^3}{3}\Big|_{AC} - \frac{10(x-4)^3}{3} - \frac{37.5(x-4)^2}{2}\Big|_{CD}$$

$$EJw = C_2 + C_1 x - \frac{2.5x^3}{6}\Big|_{BA} + \frac{10(x-2)^4}{12}\Big|_{AC} - \frac{10(x-4)^4}{12} - \frac{37.5(x-4)^3}{6}\Big|_{CD}$$

w. brzegowe 1) $w(0) = w_B = 0$
2) $w(4) = w_C = 0$

1) $0 = C_2 + C_1 \cdot 0 - 0$
2) $0 = C_2 + 4C_1 - \frac{2.5 \cdot 64}{6} + \frac{10 \cdot 16}{12}$ $\Rightarrow$ $C_2 = 0$, $C_1 = 3.33$

$$w_A = \frac{1}{EJ}\left[2 \cdot 3{,}33 - \frac{2.5 \cdot 8}{6}\right] = \frac{3.33}{EJ} \qquad w_D = \frac{80}{EJ}$$

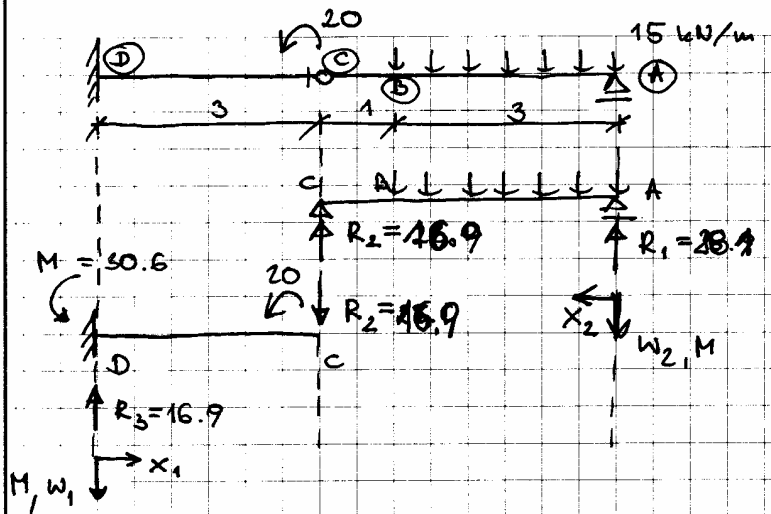
up 

$E = 200$ GPa

$$EJ = 200 \cdot 10^6 \cdot \frac{3(6)^3 \cdot 10^{-8}}{12} = 108 \text{ kNm}^2$$

$$w_A = \frac{3.33}{108}\,[m] = 3.09 \text{ cm}$$

③ Narysować ugiętą oś belki



$\Sigma M_A = 0 \quad 4R_2 - 15\cdot 3\cdot 1.5 = 0$

$R_2 = 16.9$

$\Sigma Y = 0 \quad 16.9 + R_1 - 15\cdot 3 = 0$

$R_1 = 28.1$

$\Sigma M_D = 0 \quad -M - 20 + 3\cdot 16.9 = 0$

$M = 30.6$

Belka AC

$$EJ w_2'' = -28.1 x_2 + 15\frac{x_2^2}{2}\Big|_{AB} - 15\frac{(x_2-3)^2}{2}\Big|_{BC}$$

$$EJ w_2' = C_3 - \frac{28.1 x_2^2}{2} + \frac{15 x_2^3}{6}\Big|_{AB} - \frac{15(x_2-3)^3}{6}\Big|_{BC}$$

$$EJ w_2 = C_4 + C_3 x_2 - \frac{28.1 x_2^3}{6} + \frac{15 x_2^4}{24}\Big|_{AB} - \frac{15(x_2-3)^4}{24}\Big|_{BC}$$

w. brzegowy $\quad w_2(0) = 0 \Rightarrow C_4 = 0$

Belka CD

$$EJ w_1'' = -16.9 x_1 + 30.6$$

$$EJ w_1' = C_1 - \frac{16.9 x_1^2}{2} + 30.6 x_1$$

$$EJ w_1 = C_2 + C_1 x_1 - \frac{16.9 x_1^3}{6} + \frac{30.6 x_1^2}{2}$$

w. brzegowe $\quad w_1'(0) = 0 \rightarrow C_1 = 0$

$w_1(0) = 0 \rightarrow C_2 = 0$

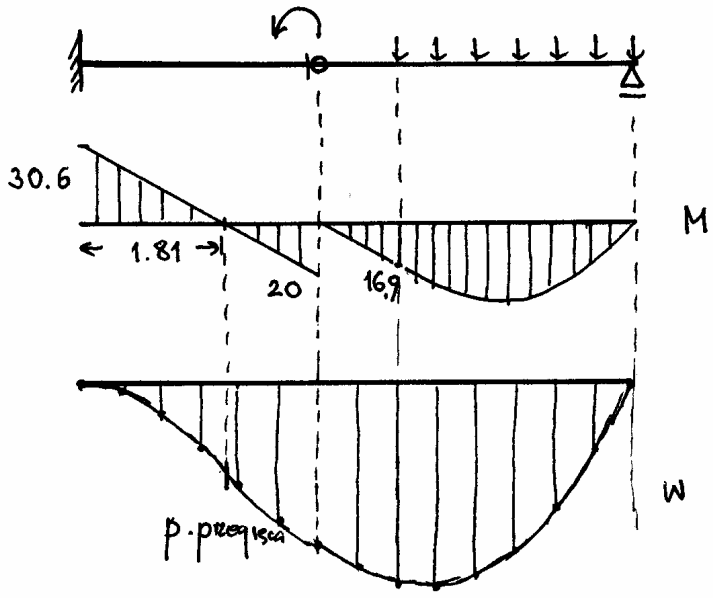
w. zszycia $\quad w_1(3) = w_2(4)$

$$-\frac{16.9\cdot 27}{6} + \frac{30.6\cdot 9}{2} = 4C_3 - \frac{28.1\cdot 64}{6} + \frac{15\cdot 16\cdot 16}{24} - \frac{15\cdot 1}{24} \Rightarrow C_3 = 50.5$$

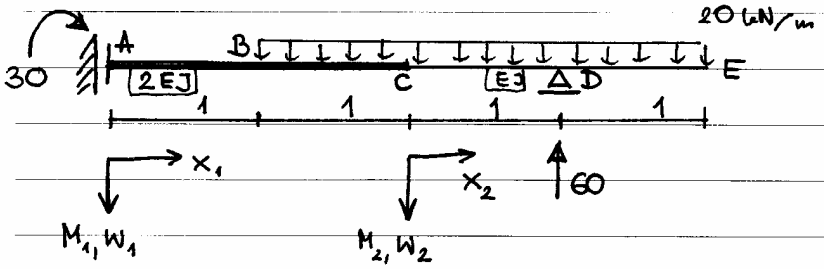
$$w_1(x_1) = \frac{1}{EJ}\left[-2{,}82 x_1^3 + 15{,}3 x_1^2\right]$$

$$w_2(x_2) = \frac{1}{EJ}\left[50.5 x_2 - 4.68 x_2^3 + 0.625 x_2^4\Big|_{AB} - 0.625(x_2-3)^4\Big|_{BC}\right]$$

x_1	0	0,5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
EJw	0	3.47	12.5	24.9	38.6	51.6	61.6	69.8	75.8	77,5	73.6	63.1	46.4	24,7	0


30.6
1.81
20
16,9
M
W
p. przęsła

④ Narysować ugiętą oś belki



Belka AC

$$M(x_1) = 30|_{AB} - \frac{20(x_1-1)^2}{2}|_{BC}$$

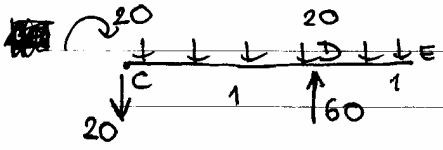
$$2EJ\, w_1'' = -30|_{AB} + 10(x_1-1)^2|_{BC}$$

$$2EJ\, w_1' = C_1 - 30x_1|_{AB} + \frac{10(x_1-1)^3}{3}|_{BC}$$

$$2EJ\, w_1 = C_2 + C_1 x_1 - 15x_1^2|_{AB} + \frac{10}{12}(x_1-1)^4|_{BC}$$

w. brzegowy $w_1'(0) = 0 \Rightarrow \boxed{C_1 = 0}$

Belka CE



$$M(x_2) = 20 - 20x_2 - 20x_2^2/2|_{CD} + 60(x_2-1)|_{DE}$$

$$EJ\, w_2'' = -20 + 20x_2 + 10x_2^2|_{CD} - 60(x_2-1)|_{DE}$$

$$EJ\, w_2' = C_3 - 20x_2 + \frac{20}{2}x_2^2 + \frac{10}{3}x_2^3|_{CD} - 30(x_2-1)^2|_{DE}$$

$$EJ\, w_2 = C_4 + C_3 x_2 - 10x_2^2 + \frac{10}{3}x_2^3 + \frac{10}{12}x_2^4|_{CD} - 10(x_2-1)^3|_{DE}$$

w. brzegowy $w(1) = w_D = 0 \Rightarrow 0 = C_4 + C_3 - 10 + 3.33 + 0.833$

$$C_3 + C_4 = 5.833$$

war. zszycia:

$w_1(2) = w_2(0) \Rightarrow \frac{1}{2EJ}[C_2 - 60 + 0.833] = \frac{1}{EJ}[C_4] \Rightarrow \boxed{C_4 = 0{,}5C_2 - 29.58}$

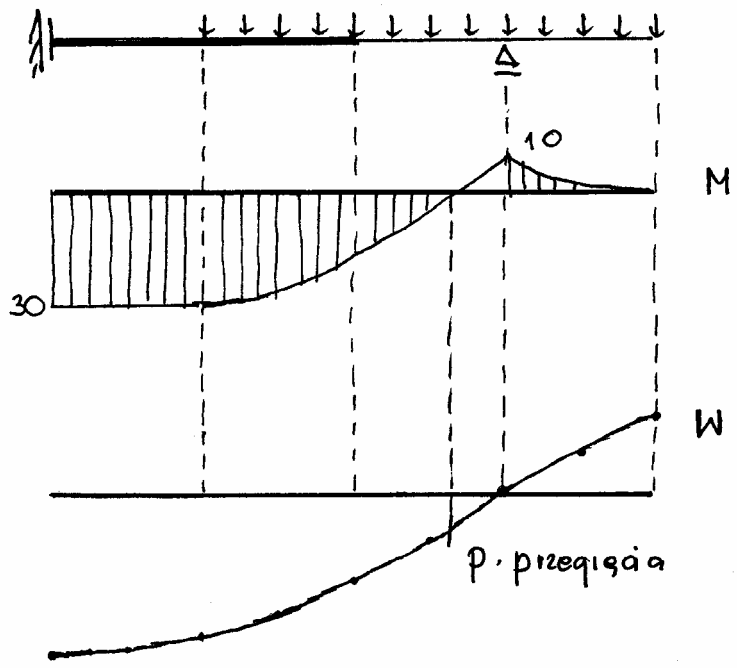
$w_1'(2) = w_2'(0) \Rightarrow \frac{1}{2EJ}[-60 + 3.333] = \frac{1}{EJ}[C_3] \Rightarrow \boxed{C_3 = -28.333}$

$\boxed{C_4 = 34.17}$ $\boxed{C_2 = 127.5}$

$$EJ\, w_1 = 63.75 - 7.5x_1^2|_{AB} + 0.417(x_1-1)^4|_{BC}$$

$$EJ\, w_2 = 34.17 - 28.33x_2 - 10x_2^2 + 3.33x_2^3 + 0.833x_2^4|_{CD} - 10(x_2-1)^3|_{DE}$$

x_1	0	0.5	1	1.5	2	2.5	3	3.5	4
w	63.75	61.9	56.3	46.9	34.2	18.0	0	−16.6	−32,5


10
M
30
W
p. przegięcia