

## ★ całka ogólna równania jednorodnego

$$\frac{\partial u_1}{\partial x_1} = 0 \Rightarrow u_1^0 = u_1^0(x_2, x_3)$$

$$\frac{\partial u_2}{\partial x_2} = 0 \Rightarrow u_2^0 = u_2^0(x_1, x_3)$$

$$\frac{\partial u_3}{\partial x_3} = 0 \Rightarrow u_3^0 = u_3^0(x_1, x_2)$$

$$(*) \quad \begin{array}{l} \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} = 0 \\ \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} = 0 \\ \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} = 0 \end{array} \quad \left| \begin{array}{l} \frac{\partial}{\partial x_3} \\ \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{array} \right.$$

$$\frac{\partial^2 u_1(x_2, x_3)}{\partial x_2 \partial x_3} + \frac{\partial^2 u_2(x_1, x_3)}{\partial x_1 \partial x_3} = 0 \quad (1)$$

$$\frac{\partial^2 u_2(x_1, x_3)}{\partial x_1 \partial x_3} + \frac{\partial^2 u_3(x_1, x_2)}{\partial x_1 \partial x_2} = 0 \quad (2)$$

$$\frac{\partial^2 u_1(x_2, x_3)}{\partial x_2 \partial x_3} + \frac{\partial^2 u_3(x_1, x_2)}{\partial x_1 \partial x_2} = 0 \quad (3)$$

$$(1) - (2) + (3) \Rightarrow \frac{\partial^2 u_1(x_2, x_3)}{\partial x_2 \partial x_3} = 0 \Rightarrow u_1^0(x_2, x_3) = a + b x_2 + c x_3 \quad (4)$$

$$(1) + (2) - (3) \Rightarrow \frac{\partial^2 u_2(x_1, x_3)}{\partial x_1 \partial x_3} = 0 \Rightarrow u_2^0(x_1, x_3) = d + e x_1 + f x_3 \quad (5)$$

$$(1) - (2) - (3) \Rightarrow \frac{\partial^2 u_3(x_1, x_2)}{\partial x_1 \partial x_2} = 0 \Rightarrow u_3^0(x_1, x_2) = g + h x_1 + j x_2 \quad (6)$$

Po wstawieniu (4), (5), (6) do równań (\*) otrzymujemy  $e = -b$ ,  $j = -f$ ,  $h = -c$

$$u_1^0(x_2, x_3) = a + b x_2 + c x_3$$

$$u_2^0(x_1, x_3) = d - b x_1 + f x_3$$

$$u_3^0(x_1, x_2) = g - c x_1 - f x_2$$